Characteristics of Fluids

- Gas or liquid both are fluid
- "Large" molecular spacing relative to a solid
- "Weak" intermolecular cohesive forces
- Can not resist a shear stress in a stationary state
- Will take the shape of its container
- Generally considered a continuum
- Viscosity distinguishes different types of fluids

Fluid Mechanics Overview

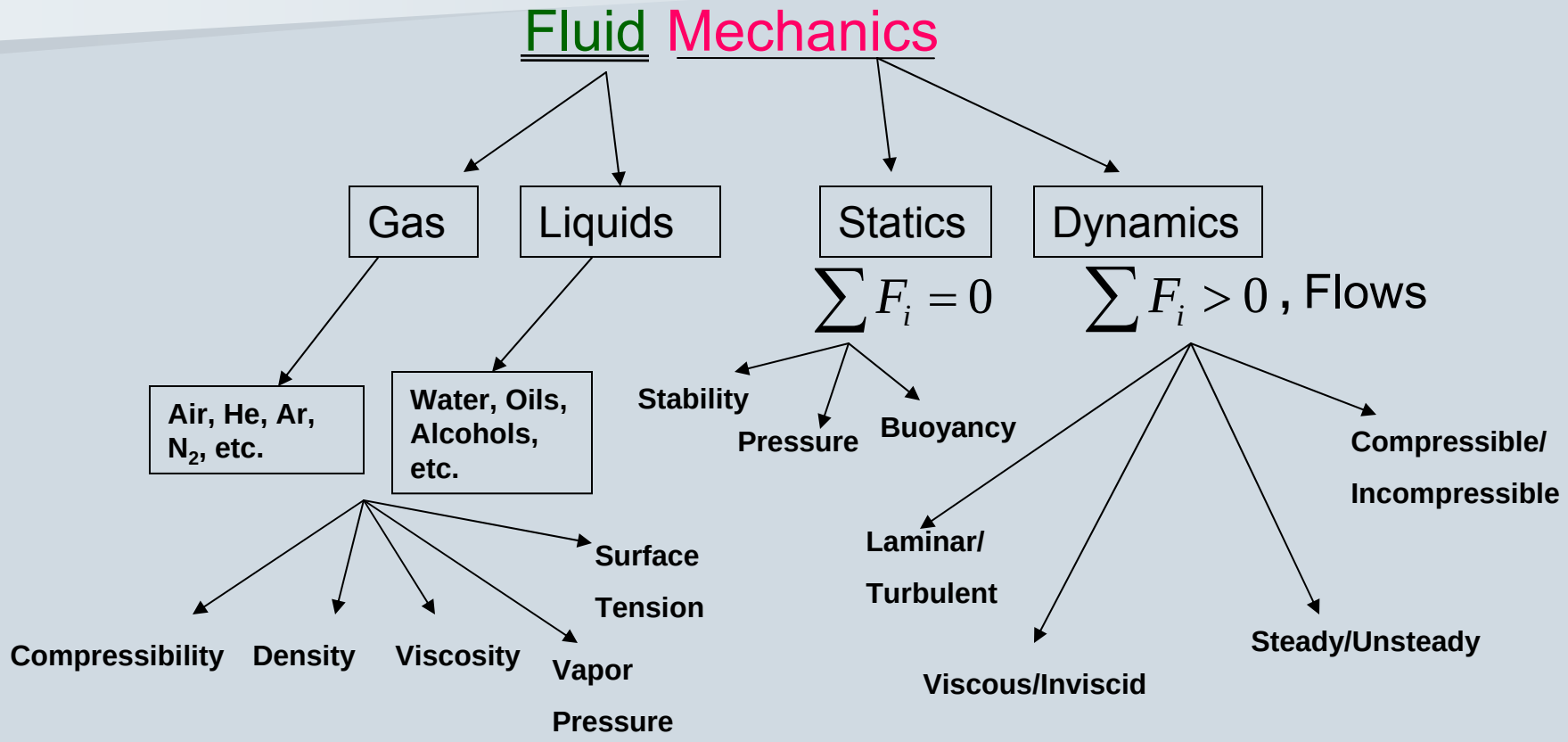
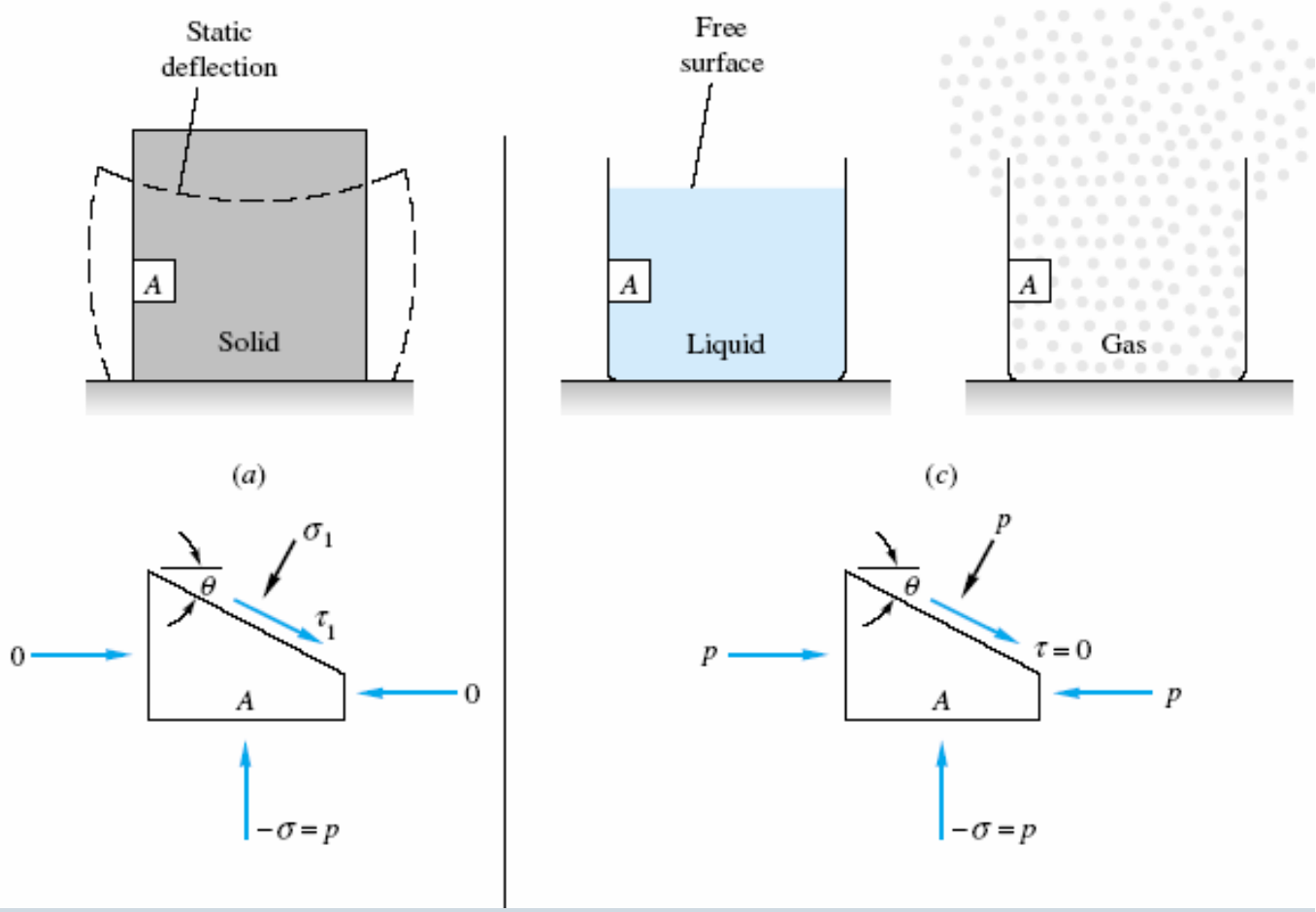


Fig. 1.1 A solid at rest can resist shear. (*a*) Static deflection of the solid; (*b*) equilibrium and Mohr's circle for solid element *A*. A fluid cannot resist shear. (*c*) Containing walls are needed; (*d*) equilibrium and Mohr's circle for fluid element *A*.

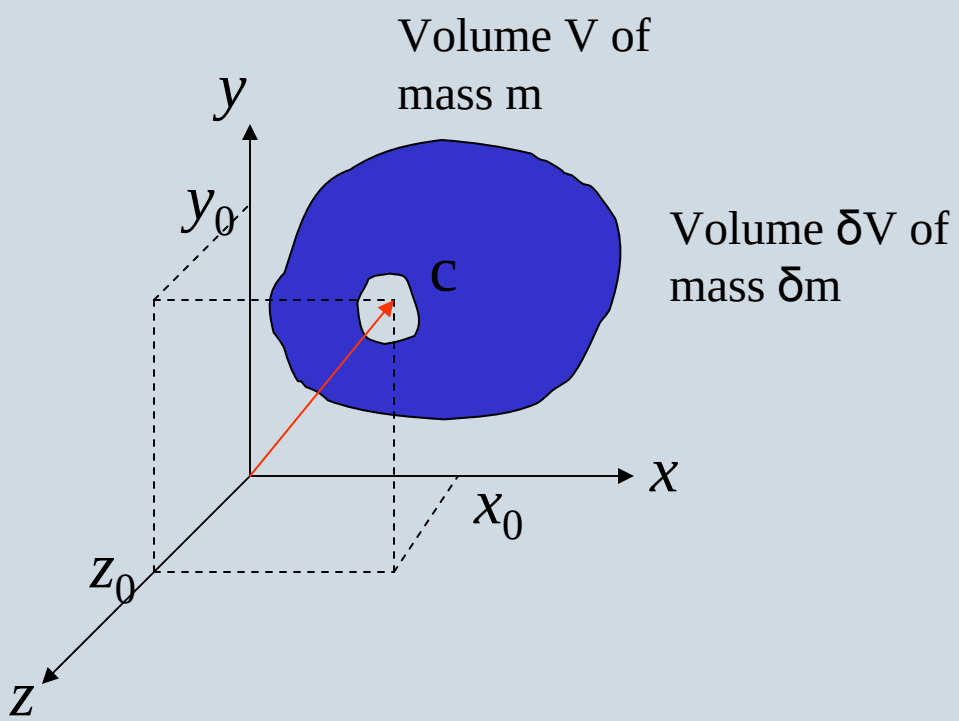


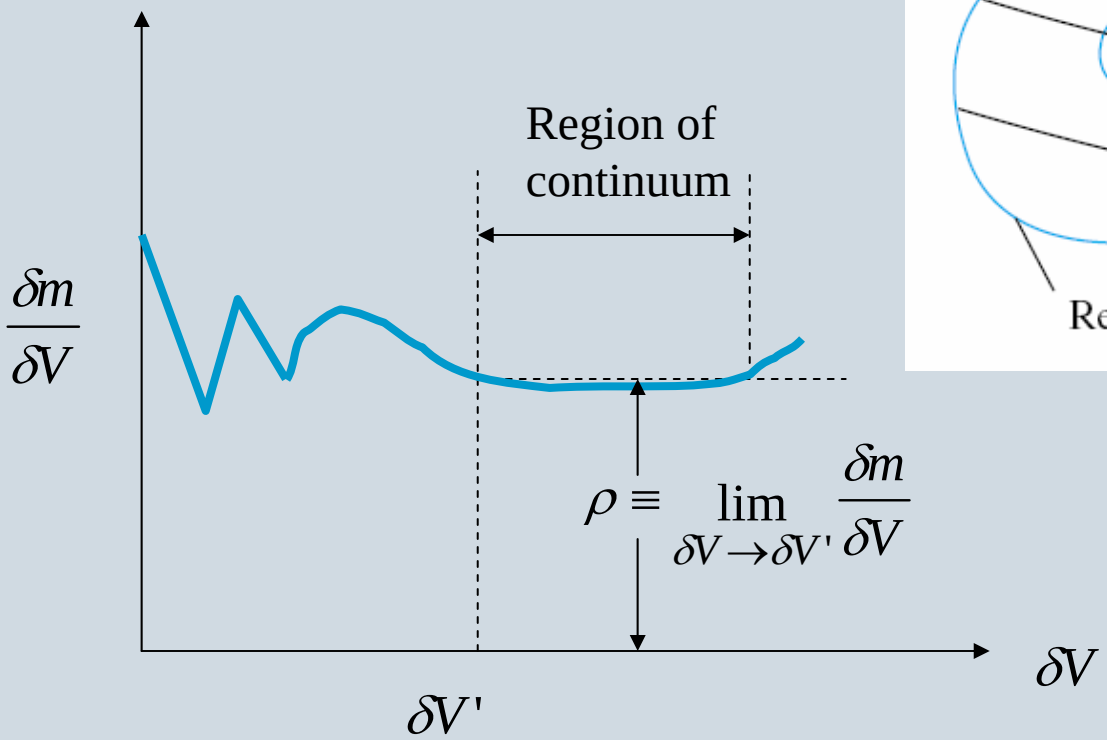
Fluid As a Continuum

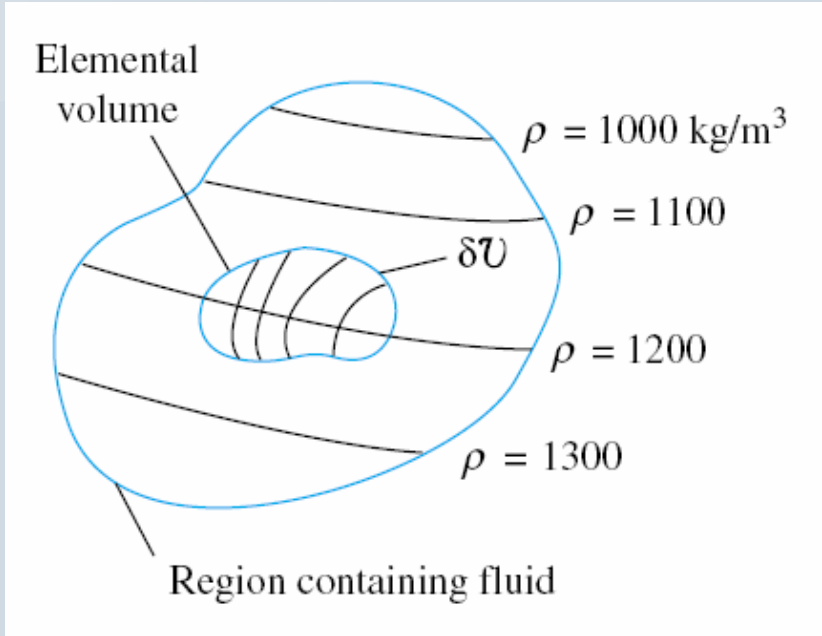
- Fluid is treated as an infinitely dividable substance.
- Each fluid property is assumed to have a definite value at every point in space.
- The fluid properties are continuous function of position and time.

e.g. density at a “point”

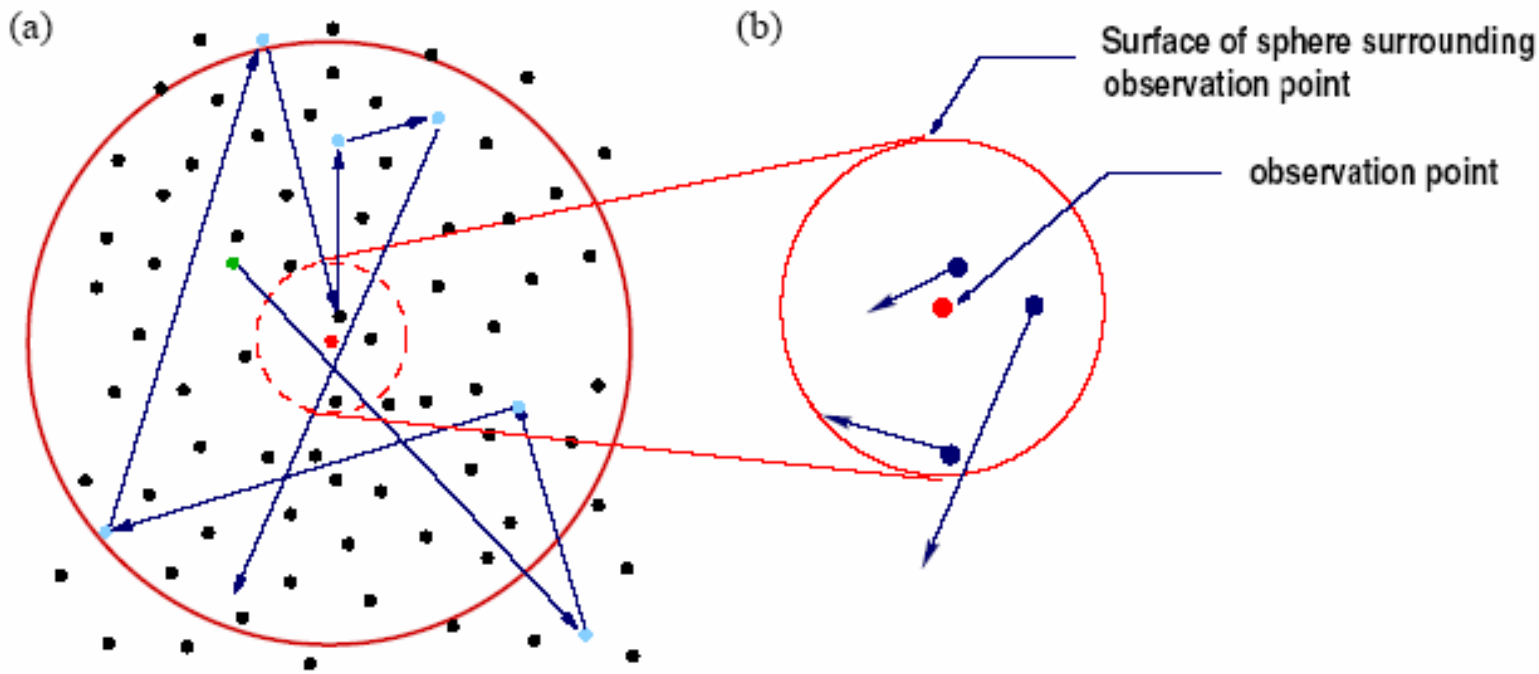
$$\rho \equiv \lim_{\delta V \to \delta V'} \frac{\delta m}{\delta V} = \rho(x, y, z, t)$$




Region of continuum
$\frac{\delta m}{\delta V}$
$\rho \equiv \lim_{\delta V \to \delta V'} \frac{\delta m}{\delta V}$
$\delta V'$
δV


Elemental volume
ρ = 1000 kg/m^3
ρ = 1100
$\delta \mho$
ρ = 1200
ρ = 1300
Region containing fluid

mean free path determined as average of distances between collisions of green molecule with light blue molecules



Measures of Fluid Mass and Weight: Density

The density of a fluid is defined as mass per unit volume.

$$\rho = \frac{m}{v}$$

m = mass, and v = volume.

- Different fluids can vary greatly in density
- Liquids densities do not vary much with pressure and temperature
- Gas densities can vary quite a bit with pressure and temperature
- Density of water at 4° C : 1000 kg/m^3
- Density of Air at 4° C : 1.20 kg/m^3

Alternatively, **Specific Volume**: $\upsilon = \frac{1}{\rho}$

Measures of Fluid Mass and Weight: Specific Weight

The specific weight of fluid is its weight per unit volume.

$$\gamma = \rho g$$

g = local acceleration of gravity, 9.807 m/s^2

- Specific weight characterizes the weight of the fluid system
- Specific weight of water at 4° C : 9.80 kN/m^3
- Specific weight of air at 4° C : 11.9 N/m^3

Measures of Fluid Mass and Weight: Specific Gravity

The specific gravity of fluid is the ratio of the density of the fluid to the density of water @ 4° C.

$$SG = \frac{\rho}{\rho_{H_2O}}$$

- Gases have low specific gravities
- A liquid such as Mercury has a high specific gravity, 13.2
- The ratio is unitless.
- Density of water at 4° C : 1000 kg/m^3

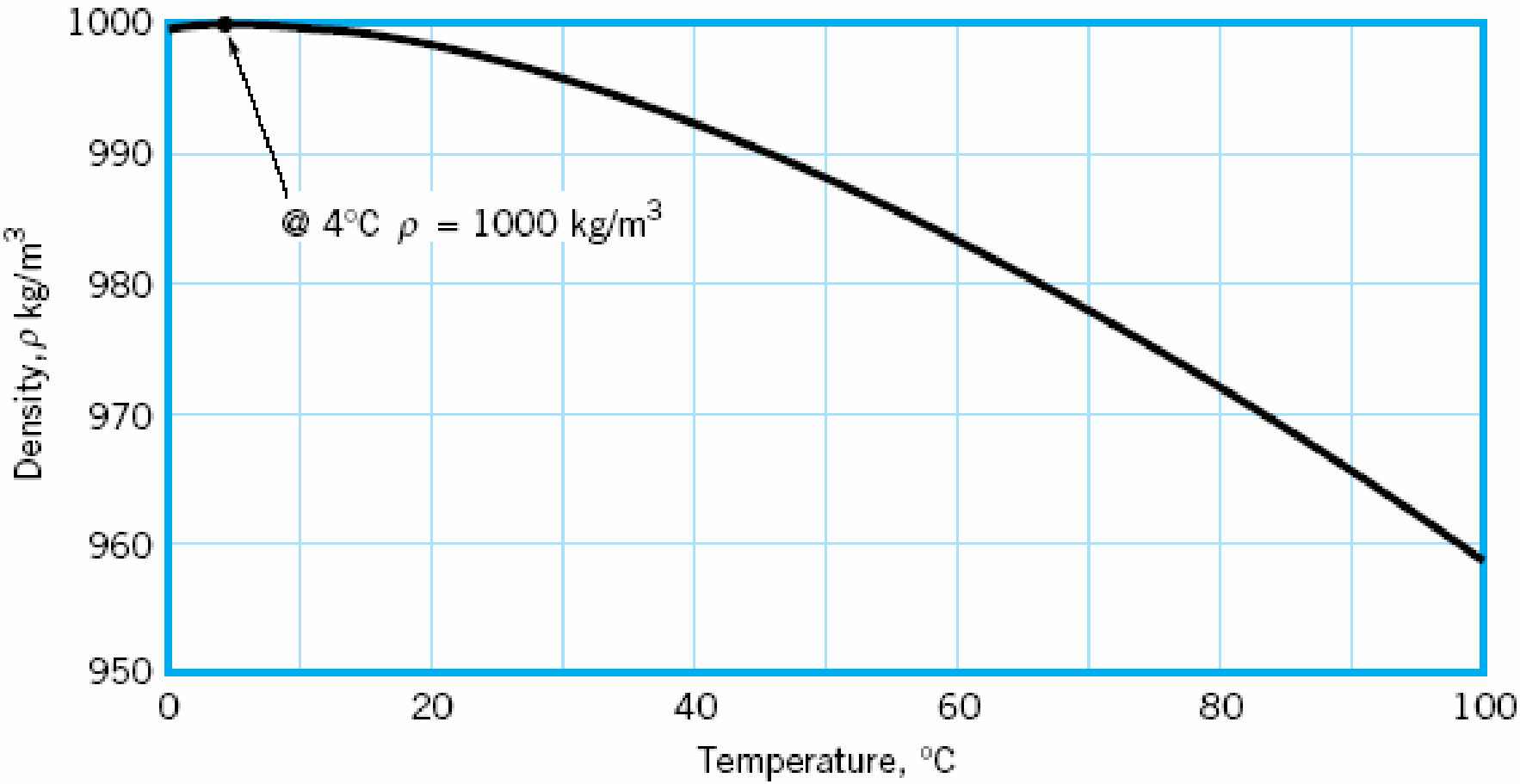


FIGURE 1.1 Density of water as a function of temperature.

The density of a liquid usually decreases slightly with temperature and increases moderately with pressure. If we neglect the temperature effect, an empirical pressure-density relation for a liquid is

$$\frac{p}{p_a} \approx (B + 1)\left(\frac{\rho}{\rho_a}\right)^n - B \tag{1.19}$$

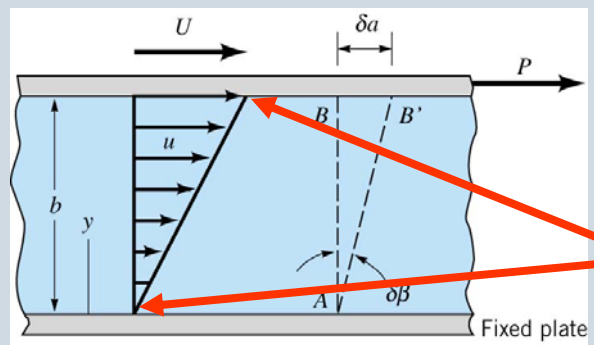
where B and n are dimensionless parameters which vary slightly with temperature and p_a and ρ_a are standard atmospheric values. Water can be fitted approximately to the values $B \approx 3000$ and $n \approx 7$.

Viscosity

The viscosity is measure of the "fluidity" of the fluid which is not captured simply by density or specific weight. A fluid can not resist a shear and under shear begins to flow. The shearing stress and shearing strain can be related with a relationship of the following form for common fluids such as water, air, oil, and gasoline:

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$

μ is the absolute viscosity or dynamics viscosity of the fluid, u is the velocity of the fluid and y is the vertical coordinate as shown in the schematic below:



"No Slip Condition"

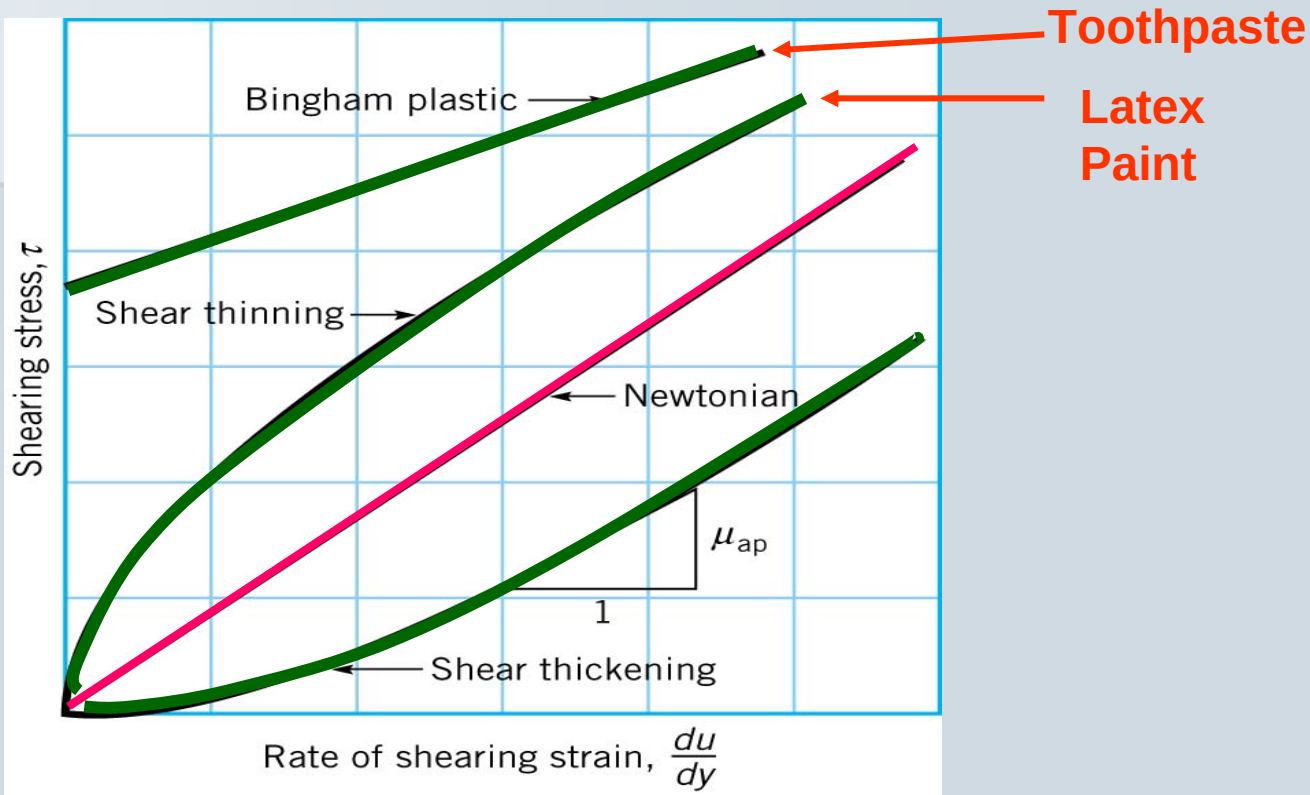
Viscosity: Measurements

- Viscometer is used for measuring the viscosity of the fluid.

- Viscosity Varies from Fluid to Fluid and is dependent on temperature, thus temperature is measured as well.

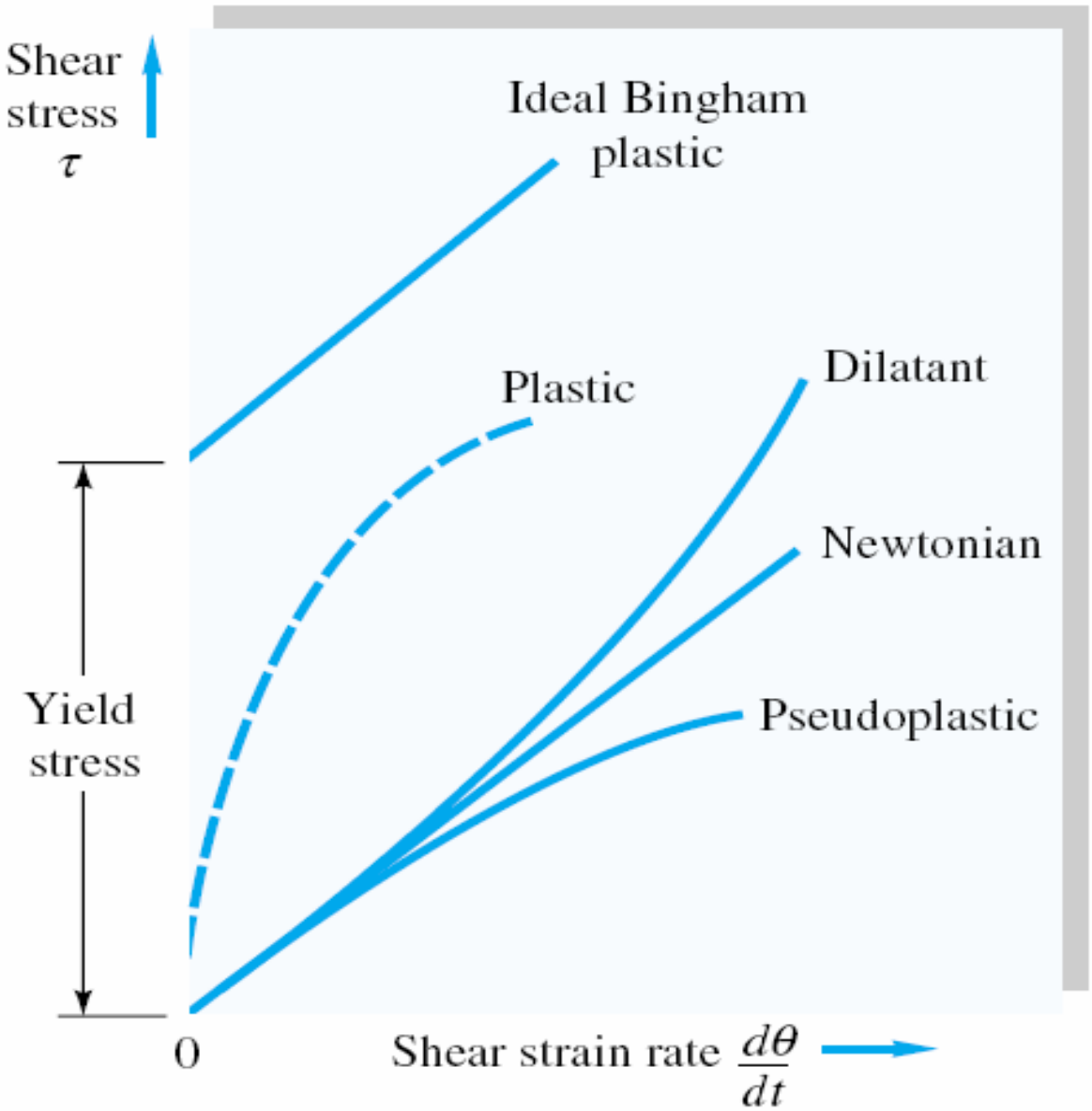
- Units of Viscosity are $N \cdot s/m^2$ or $lb \cdot s/ft^2$

Viscosity: Newtonian vs. Non-Newtonian

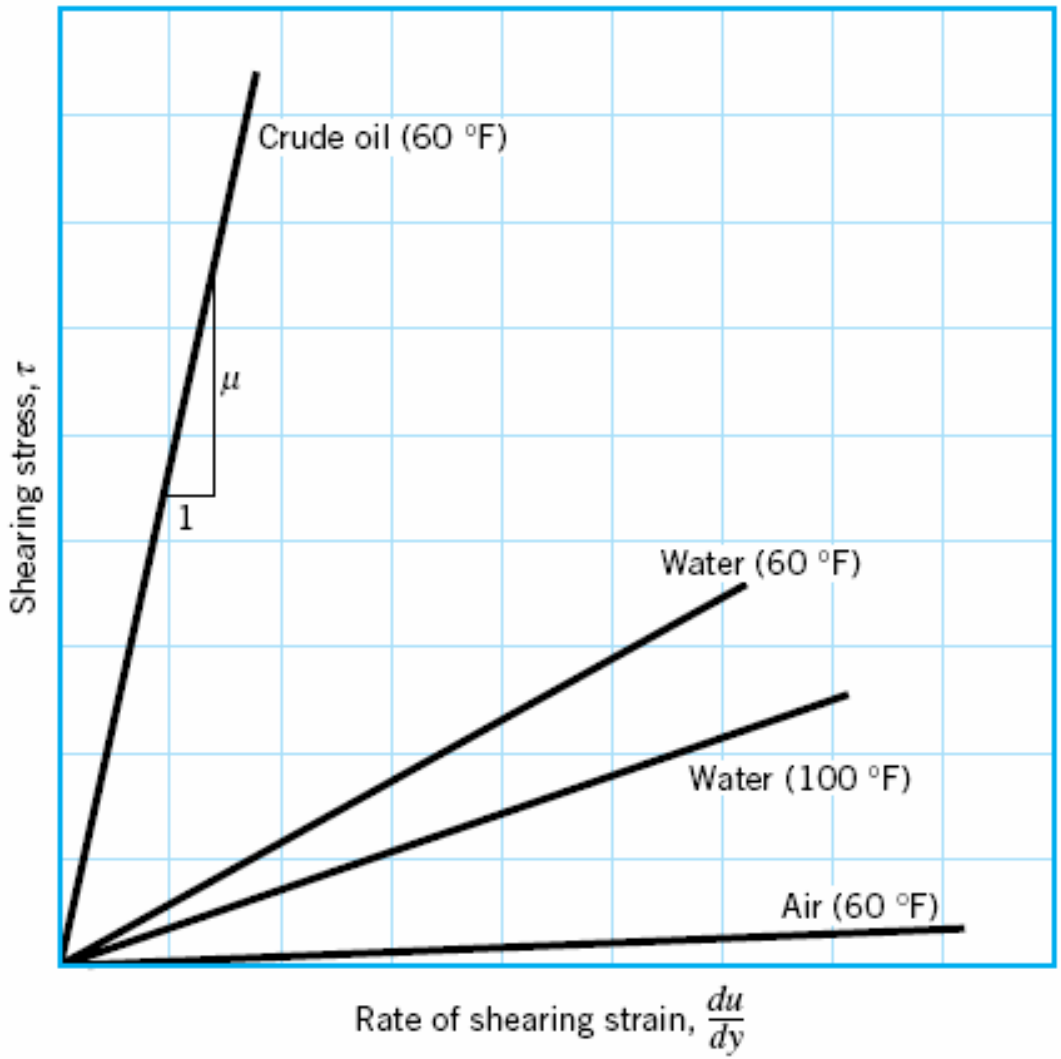


Newtonian Fluids are Linear Relationships between stress and strain: Most common fluids are Newtonian.

Non-Newtonian Fluids are Non-Linear between stress and strain

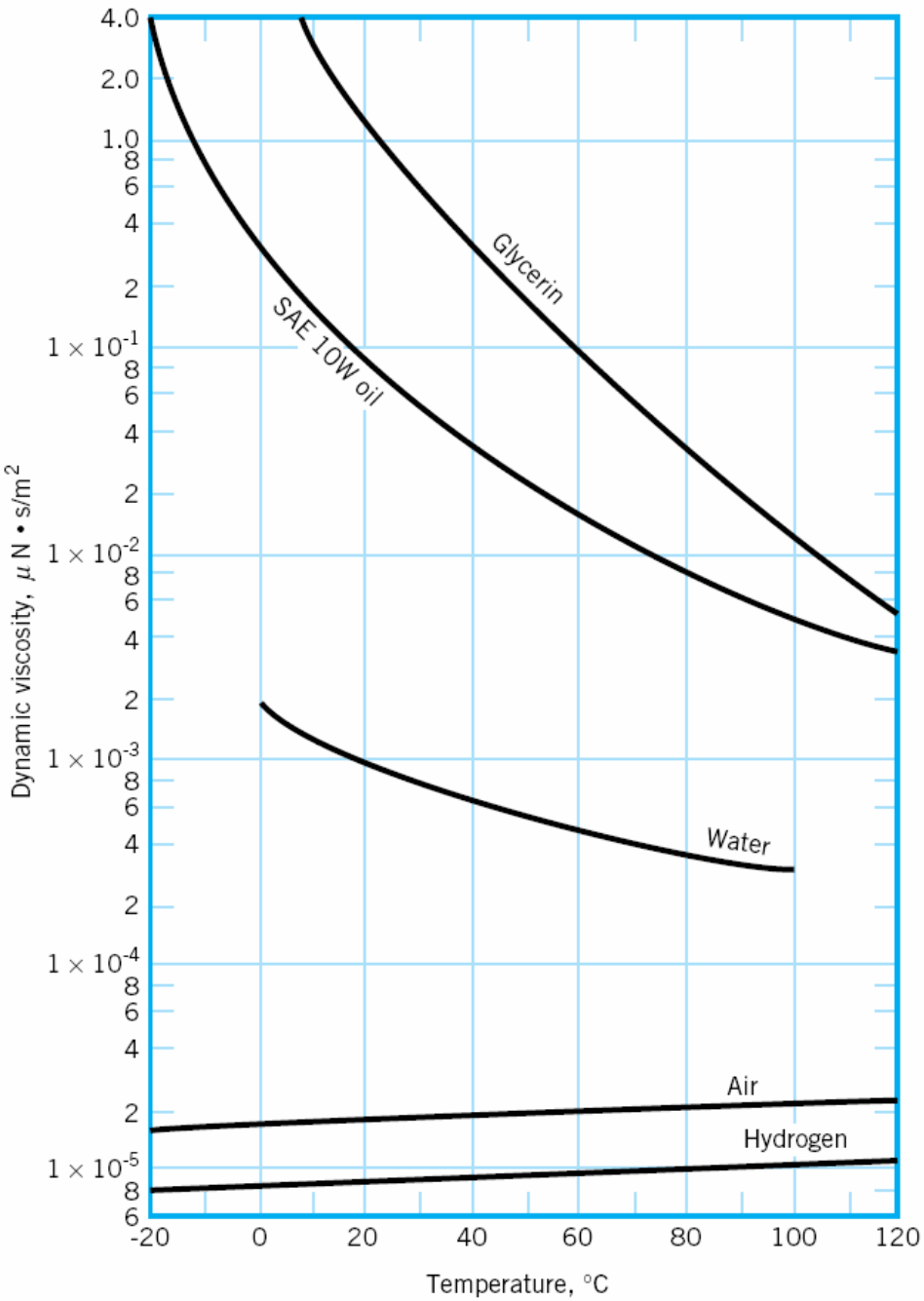


$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$



■ **FIGURE 1.4** Linear variation of shearing stress with rate of shearing strain for common fluids.

T=١5.5, 15.5, 38, 15.5



■ FIGURE 1.6
Dynamic (absolute) viscosity of some common fluids as a function of temperature.

Table 2.3 Viscosity and kinematic viscosity of water and air at standard atmospheric pressure

	Water		Air	
Temp. (°C)	**Viscosity, μ (Pa s $\times 10^5$)**	**Kinematic viscosity, ν ($m^2/s \times 10^6$)**	**Viscosity, μ (Pa s $\times 10^5$)**	**Kinematic viscosity, ν ($m^2/s \times 10^6$)**
0	179.2	1.792	1.724	13.33
10	130.7	1.307	1.773	14.21
20	100.2	1.004	1.822	15.12
30	79.7	0.801	1.869	16.04
40	65.3	0.658	1.915	16.98

$$[\mu] = \left[\frac{ML/T^2}{L^2}\cdot T\right] = \left[\frac{M}{LT}\right]$$ ← (international system SI unit: M-L-T)

$\boxed{\text{Denote}}$: $\frac{N}{M^2} \equiv$ Pa, then

$$\begin{cases} \mu_{water,20°c} = 1.01\times 10^3\, Pa\cdot \sec \\ \mu_{water,100°c} = 283\, Pa\cdot \sec \end{cases}$$

(liquid): T ↗ → μ ↘

$$\begin{cases} \mu_{air,20°c} = 17.9\, Pa\cdot \sec \\ \mu_{air,100°c} = 22.9\, Pa\cdot \sec \end{cases}$$

(gas): T ↗ → μ ↗

$$\mathbf{1\ P = 100\ cP = 0.1\ Pa\ s}$$

Viscosity: Kinematic Viscosity

$$\nu = \frac{\mu}{\rho}$$

- Kinematic viscosity is another way of representing viscosity
- Used in the flow equations
- The units are of L^2/T or m^2/s and ft^2/s

The effect of temperature on viscosity can be closely approximated using two empirical formulas. For gases the *Sutherland equation* can be expressed as

$$\mu = \frac{CT^{3/2}}{T + S} \tag{1.10}$$

where C and S are empirical constants, and T is absolute temperature. Thus, if the viscosity is known at two temperatures, C and S can be determined. Or, if more than two viscosities are known, the data can be correlated with Eq. 1.10 by using some type of curve-fitting scheme.

For liquids an empirical equation that has been used is

$$\mu = De^{B/T} \tag{1.11}$$

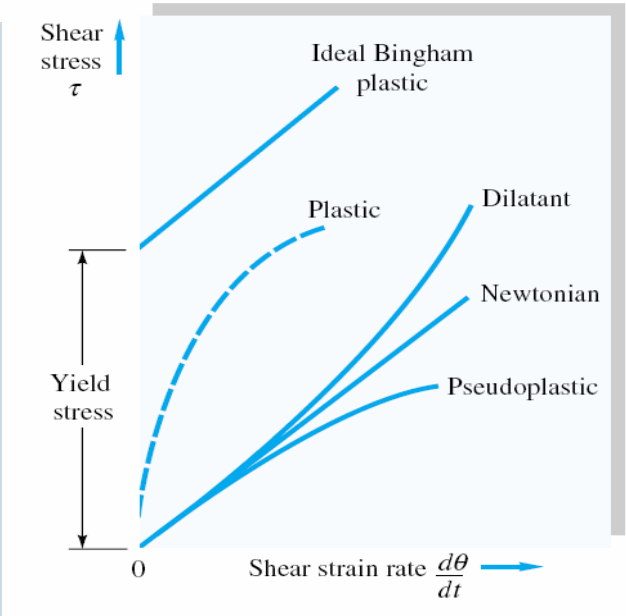
where D and B are constants and T is absolute temperature. This equation is often referred to as *Andrade's equation.* As was the case for gases, the viscosity must be known at least for two temperatures so the two constants can be determined. A more detailed discussion of the effect of temperature on fluids can be found in Ref. 1.

Non-Newtonian Fluids

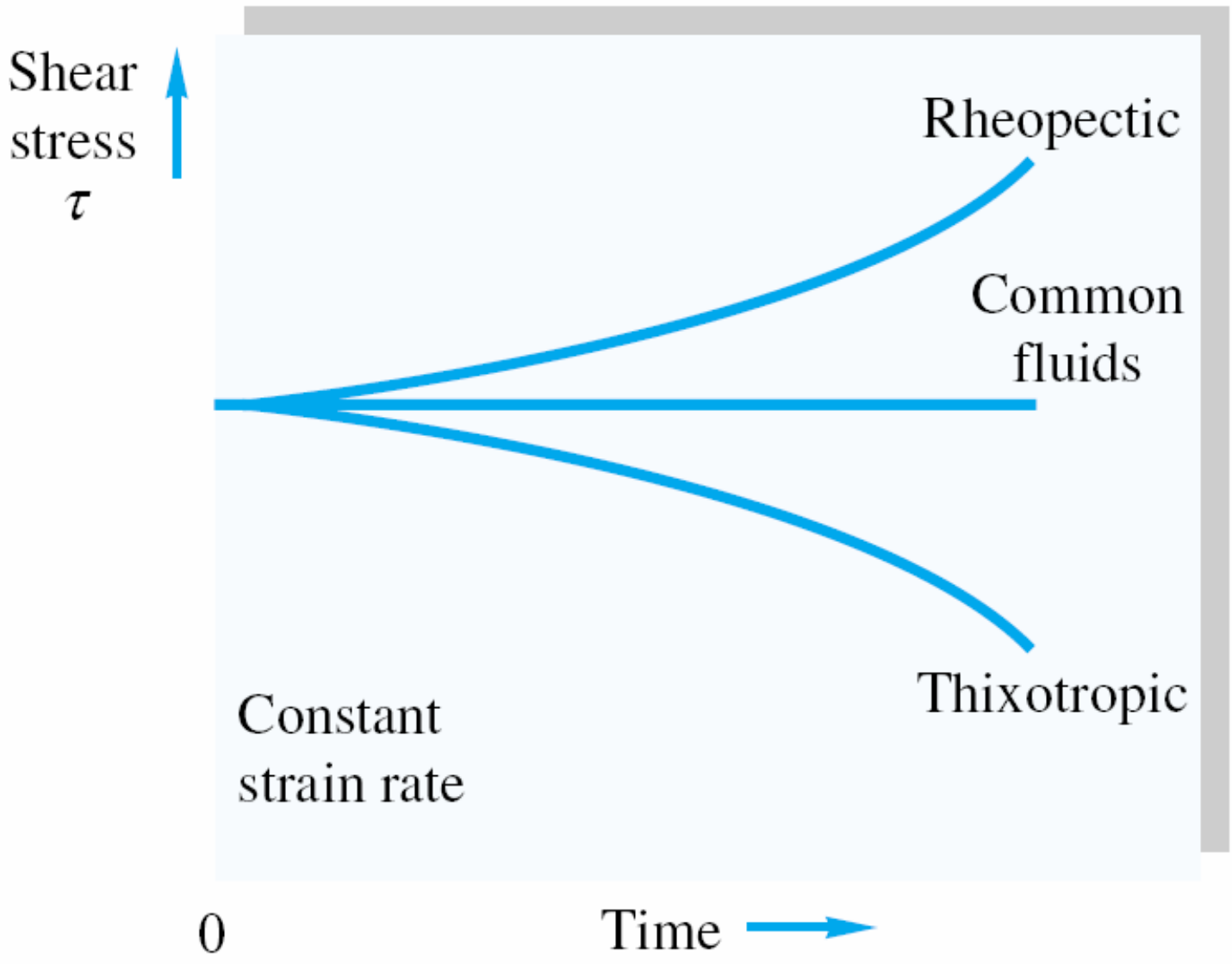
It is common for the shear stress of non-Newtonian fluids to be expressed in terms of an empirical relation of the form

$$\tau = \mathcal{K}\left(\frac{du}{dy}\right)^n ,$$

where the exponent n is called the *flow behavior index*, and $\mathcal{K}$ is termed the *consistency index*. This representation is often called a "power law," and fluids whose shear stress can be accurately represented in this way are called *power-law fluids*. For more information on such fluids the reader is referred to more advanced texts and monographs on fluid dynamics.

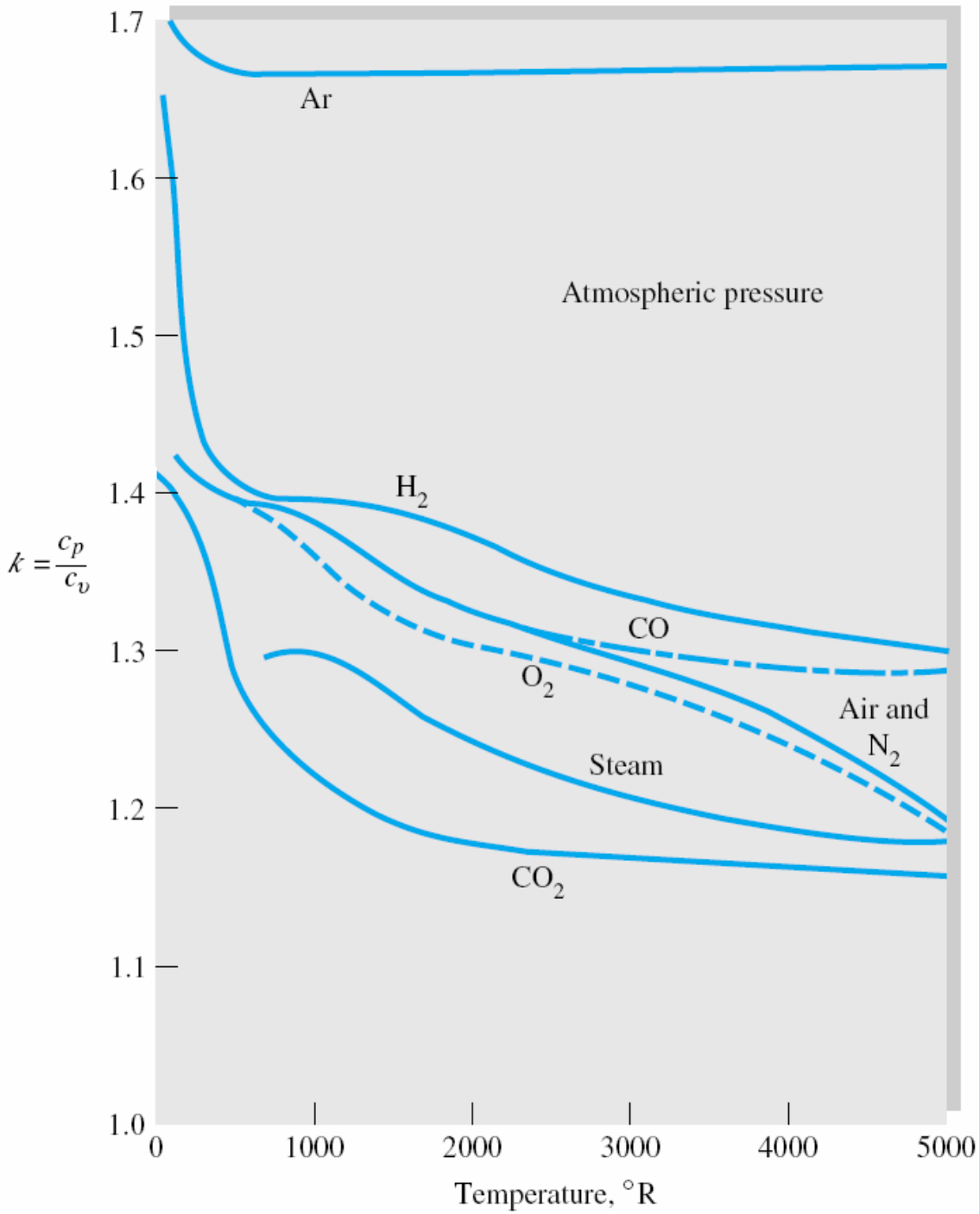


Time effects on viscosity



Rheology

<table>
<tr><td rowspan="4">Continuum mechanics</td><td rowspan="2">Solid mechanics or strength of materials</td><td colspan="2">Elasticity</td></tr>
<tr><td>Plasticity</td><td rowspan="3">Rheology</td></tr>
<tr><td rowspan="2">Fluid mechanics</td><td>Non-Newtonian fluids</td></tr>
<tr><td colspan="2">Newtonian fluids</td></tr>
</table>


1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
$k = \frac{c_p}{c_v}$
Ar
Atmospheric pressure
H_2
CO
O_2
Air and
N_2
Steam
CO_2
0
1000
2000
3000
4000
5000
Temperature, °R

Compressibility of Fluids: Bulk Modulus

$$E_{\upsilon} = \frac{dp}{d\rho / \rho}$$

P is pressure, and ρ is the density.

- Measure of how pressure compresses the volume/density
- Units of the bulk modulus are N/m^2 (Pa) and $lb/in.^2$ (psi).
- Large values of the bulk modulus indicate incompressibility
- Incompressibility indicates large pressures are needed to compress the volume slightly
- It takes 213 atm to compress water 1% at atmospheric pressure and 15.5°C.
- Most liquids are incompressible for most practical engineering problems.

$$\beta = 1/K$$

Compressibility of Fluids: Compression of Gases

Ideal Gas Law: $p = \rho RT$

P is pressure, ρ is the density, R is the gas constant, and T is Temperature

Isothermal Process (constant temperature):

$$\frac{p}{\rho} = cons\tan t \quad \Rightarrow \quad E_\upsilon = p$$

Isentropic Process (frictionless, no heat exchange):

$$\frac{p}{\rho^k} = cons\tan t \quad \Rightarrow \quad E_\upsilon = kp$$

k is the ratio of specific heats, c_p (constant pressure) to c_v (constant volume), and $R = c_p - c_v$.

If we consider air under at the same conditions as water, we can show that air is 15,000 times more compressible than water. However, many engineering applications allow air to be considered incompressible.

Table 2.5 Gas constant R and ratio of specific heat κ

Gas	Symbol	Density (kg/m^3) (0°C, 760 mm Hg)	R (SI) $m^2/(s^2\ K)$	$\kappa = c_p/c_v$
Helium	He	0.178 5	2 078.1	1.66
Air	–	1.293	287.1	1.402
Carbon monoxide	CO	1.250	296.9	1.400
Oxygen	O_2	0.089 9	4 124.8	1.409
Hydrogen	H_2	1.429	259.8	1.399
Carbon dioxide	CO_2	1.977	189.0	1.301
Methane	CH_4	0.717	518.7	1.319

Compressibility of Fluids: Speed of Sound

A consequence of the compressibility of fluids is that small disturbances introduced at a point propagate at a finite velocity. Pressure disturbances in the fluid propagate as sound, and their velocity is known as the speed of sound or the acoustic velocity, c.

$$c = \sqrt{\frac{dp}{d\rho}} \quad or \quad c = \sqrt{\frac{E_v}{\rho}}$$

<u>Isentropic Process (frictionless, no heat exchange because):</u>

$$c = \sqrt{\frac{kp}{\rho}}$$

<u>Ideal Gas and Isentropic Process:</u>

$$c = \sqrt{kRT}$$

Compressibility of Fluids: Speed of Sound

- Speed of Sound in Air at 60 °F ≈ 1117 ft/s or 300 m/s
- Speed of Sound in Water at 60 °F ≈ 4860 ft/s or 1450 m/s
- If a fluid is truly incompressible, the speed of sound is infinite, however, all fluids compress slightly.

Example: A jet aircraft flies at a speed of 250 m/s at an altitude of 10,700 m, where the temperature is -54 °C. Determine the ratio of the speed of the aircraft, V, to the speed of sound, c at the specified altitude. Assume k = 1.40

Ideal Gas and Isentropic Process:

$$c = \sqrt{kRT}$$

$$c = \sqrt{1.40 * (286.9 \ \ J/kgK) * 219 \ \ K}$$

$$c = 296.6 \ \ m/s$$

Compressibility of Fluids: Speed of Sound

Example (Continued):

$$Ratio = \frac{V}{c}$$

$$Ratio = \frac{250\ \ m/s}{296.6\ \ m/s}$$

$$Ratio = 0.84$$

- The above ratio is known as the Mach Number, Ma
- For Ma < 1 ***Subsonic Flow***
- For Ma > 1 ***Supersonic Flow***

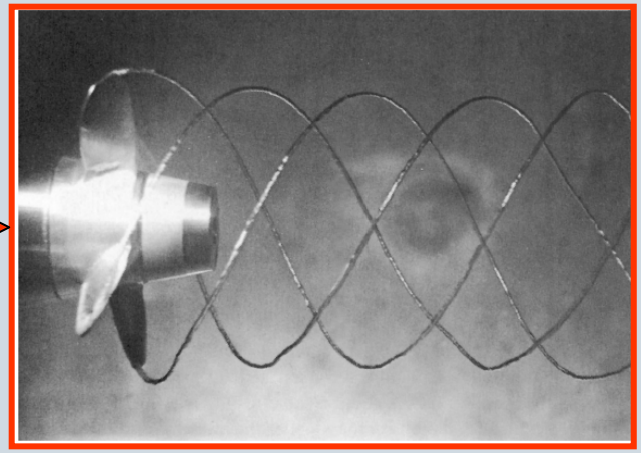
Vapor Pressure: Evaporation and Boiling

Evaporation occurs in a fluid when liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape to the atmosphere.

Vapor Pressure is that pressure exerted on the fluid by the vapor in a closed saturated system where the number of molecules entering the liquid are the same as those escaping. Vapor pressure depends on temperature and type of fluid.

Boiling occurs when the absolute pressure in the fluid reaches the vapor pressure. Boiling occurs at approximately 100 °C, but it is not only a function of temperature, but also of pressure. For example, in Colorado Spring, water boils at temperatures less than 100 °C.

Cavitation is a form of Boiling due to low pressure locally in a flow.



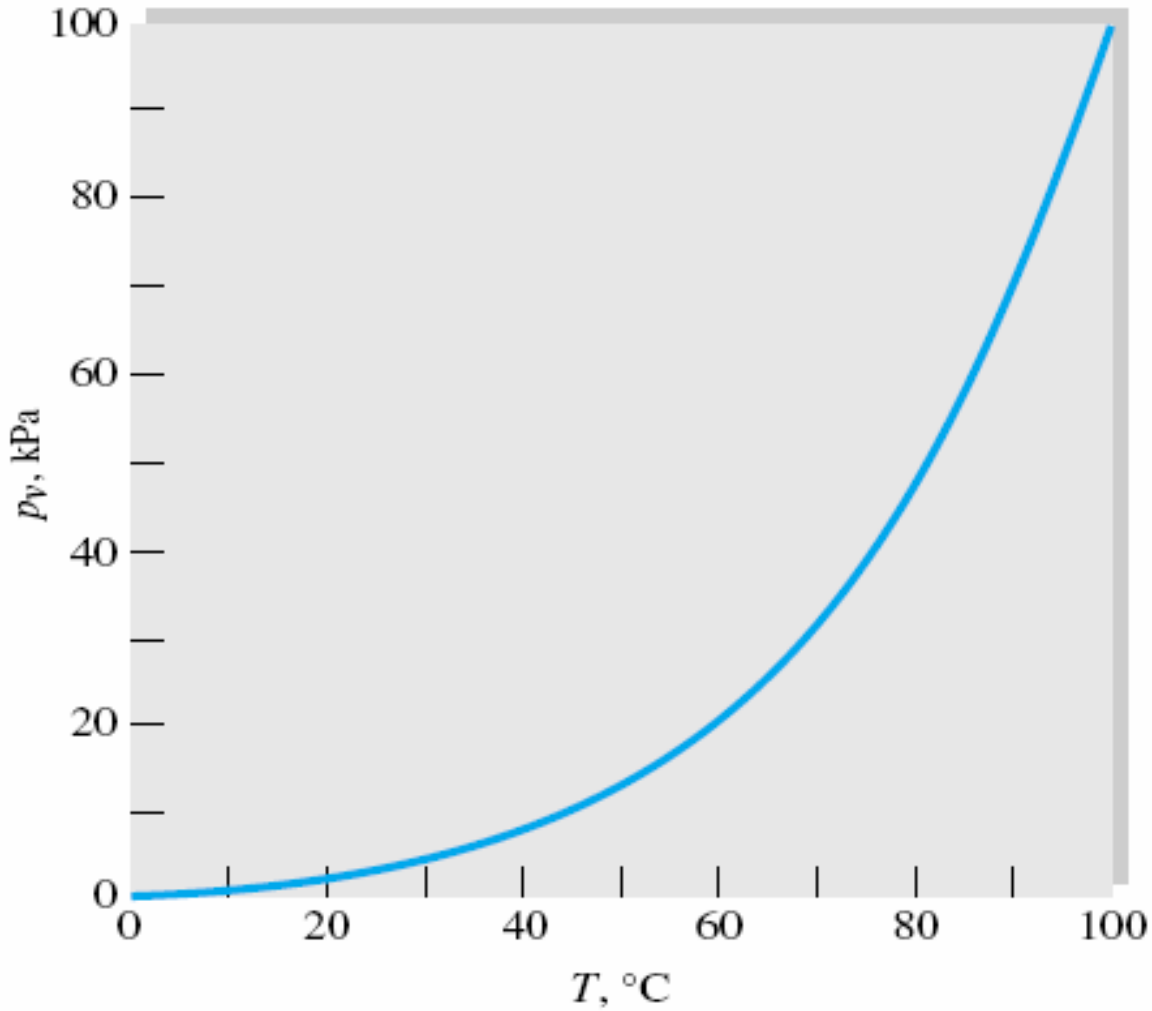


Fig. 1.11 Vapor pressure of water. Data from Table A.5.

Surface Tension

At the interface between a liquid and a gas or two immiscible liquids, forces develop forming an analogous “skin” or “membrane” stretched over the fluid mass which can support weight.

This “skin” is due to an imbalance of cohesive forces. The interior of the fluid is in balance as molecules of the like fluid are attracting each other while on the interface there is a net inward pulling force.

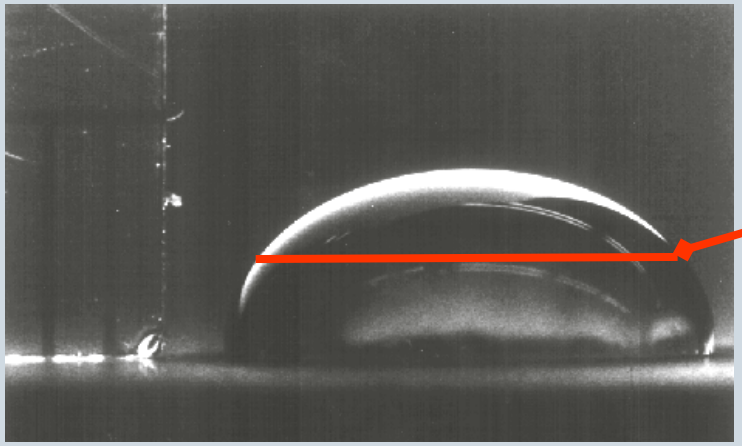
Surface tension is the intensity of the molecular attraction per unit length along any line in the surface.

Surface tension is a property of the liquid type, the temperature, and the other fluid at the interface.

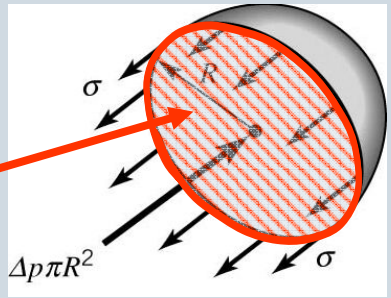
Surface Tension: Liquid Drop

The pressure inside a drop of fluid can be calculated using a free-body diagram:

Real Fluid Drops



Mathematical Model



R is the radius of the droplet, σ is the surface tension, Δp is the pressure difference between the inside and outside pressure.

The force developed around the edge due to surface tension along the line:

$$F_{surface} = 2\pi R\sigma$$

Applied to Circumference

This force is balanced by the pressure difference Δp:

$$F_{pressure} = \Delta p \pi R^2$$

Applied to Area

Surface Tension: Liquid Drop

Now, equating the Surface Tension Force to the Pressure Force, we can estimate $\Delta p = p_i - p_e$:

$$\Delta p = \frac{2\sigma}{R}$$

This indicates that the internal pressure in the droplet is greater that the external pressure since the right hand side is entirely positive.

Is the pressure inside a bubble of water greater or less than that of a droplet of water?

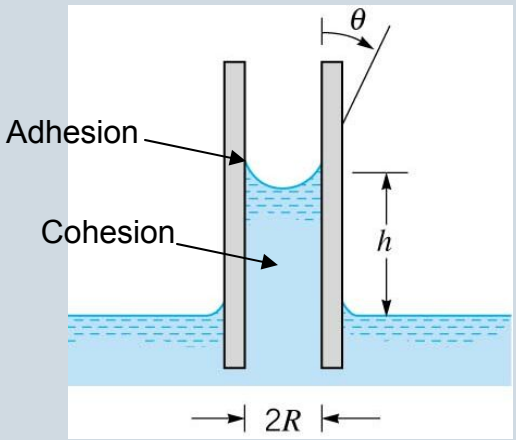
Prove to yourself the following result:

$$\Delta p = \frac{4\sigma}{R}$$

Surface Tension: Capillary Action

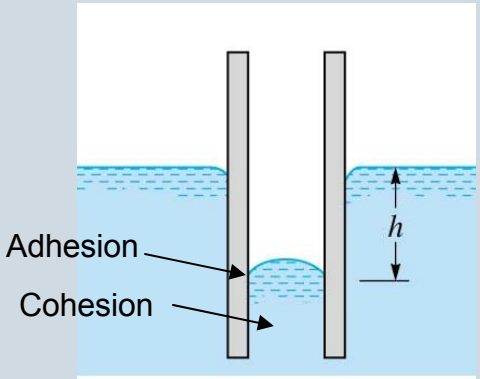
Capillary action in small tubes which involve a liquid-gas-solid interface is caused by surface tension. The fluid is either drawn up the tube or pushed down.

"Wetted"



Adhesion > Cohesion

"Non-Wetted"



Cohesion > Adhesion

h is the height, R is the radius of the tube, θ is the angle of contact.

The weight of the fluid is balanced with the vertical force caused by surface tension.

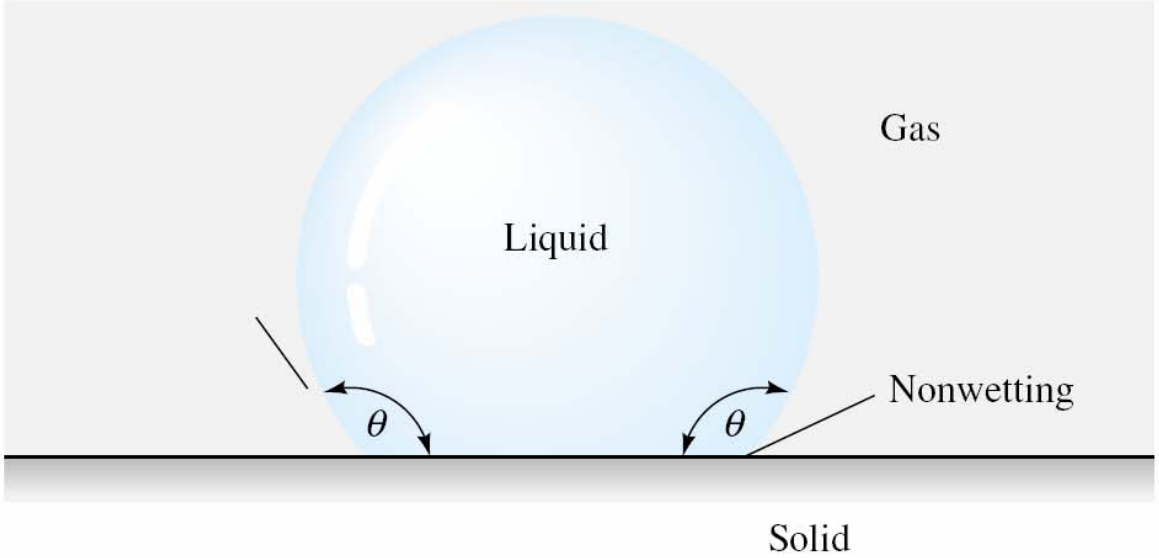


Fig. 1.10 Contact-angle effects at liquid-gas-solid interface. If $\theta < 90°$, the liquid "wets" the solid; if $\theta > 90°$, the liquid is nonwetting.

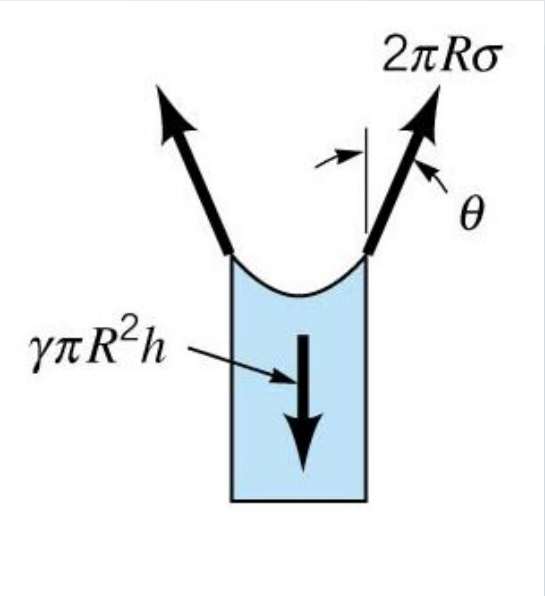
Surface Tension: Capillary Action

At what value of contact angle θ does the liquid-solid interface become "non-wetted"? $\theta > 90°$

Surface tension is apparent in many practical problems such as movement of liquid through soil and other porous media, flow of thin films, formation of drops and bubbles, and the breakup of liquid jets.

Surface Tension: Capillary Action

Free Body Diagram for Capillary Action for a Wetted Surface:



$$F_{surface} = 2\pi R\sigma\cos\theta$$

$$W = \gamma\pi R^2 h$$

Equating the two and solving for h:

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

For clean glass in contact with water, $\theta \approx 0°$, and thus as R decreases, h increases, giving a higher rise.

For a clean glass in contact with Mercury, $\theta \approx 130°$, and thus h is negative or there is a push down of the fluid.

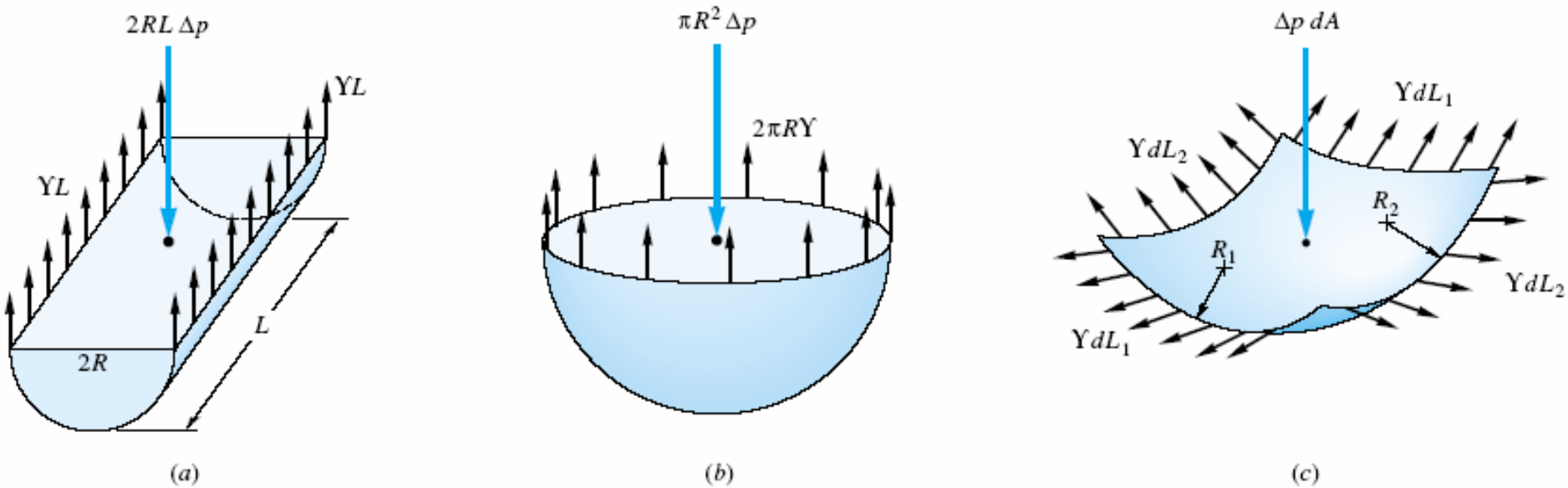


Fig. 1.9 Pressure change across a curved interface due to surface tension: (*a*) interior of a liquid cylinder; (*b*) interior of a spherical droplet; (*c*) general curved interface.

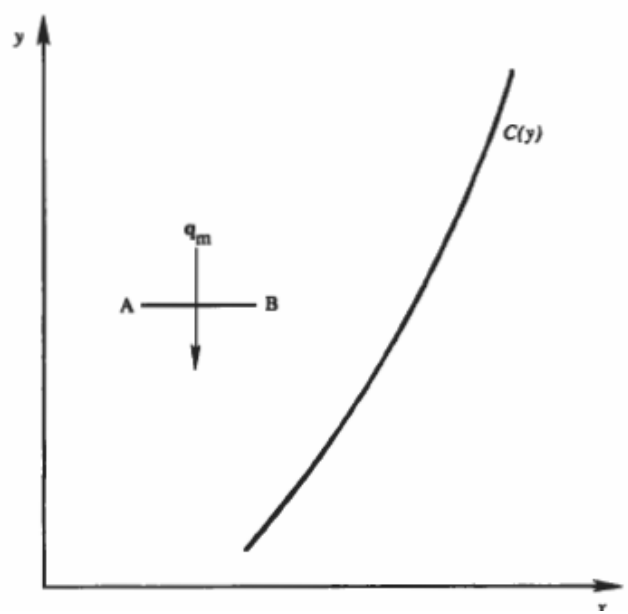
$$p_i - p_o = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

5. *Transport Phenomena*

Consider a surface area AB within a mixture of two gases, say nitrogen and oxygen (Figure 1.2), and assume that the concentration C of nitrogen (kilograms of nitrogen per cubic meter of mixture) varies across AB. Random migration of molecules across AB in both directions will result in a *net* flux of nitrogen across AB, from the region of higher C toward the region of lower C. Experiments show that, to a good approximation, the flux of one constituent in a mixture is proportional to its concentration gradient and it is given by

$$\mathbf{q}_m = -k_m \nabla C. \tag{1.1}$$

Here the vector $\mathbf{q}_m$ is the mass flux ($\mathrm{kg\,m^{-2}\,s^{-1}}$) of the constituent, ∇C is the concentration gradient of that constituent, and k_m is a constant of proportionality that



Thermal Conductivity

Just as viscosity relates applied stress to resulting strain rate, there is a property called *thermal conductivity* k which relates the vector rate of heat flow per unit area $\mathbf{q}$ to the vector gradient of temperature ∇T. This proportionality, observed experimentally for fluids and solids, is known as *Fourier's law of heat conduction*

$$\mathbf{q} = -k\nabla T \tag{1.29a}$$

which can also be written as three scalar equations

$$q_x = -k\frac{\partial T}{\partial x} \qquad q_y = -k\frac{\partial T}{\partial y} \qquad q_z = -k\frac{\partial T}{\partial z} \tag{1.29b}$$

End